

Entrainment in a Forced-Circulation Evaporator

by

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THE purpose of this research was to study the entrainment of liquor by the vapor in an evaporator of the forced-circulation type, and to determine the relation between the quantity of liquor carried by the vapor and the conditions maintained within the evaporator.

The small amount of work that has been done on the subject of entrainment has all pertained to steam boilers. A large part of this work has been done by Foulk who has also made a rather complete summary of what has been done by others (2). Vorkauf (4) studied the subject comprehensively using small-scale laboratory apparatus designed to give conditions similar to those in a steam boiler.

The work referred to is of interest to this problem only inasmuch as it gives some insight into the mechanism of entrainment and advances some theories upon the formation and behavior of the droplets. The results obtained, however, are not directly applicable because the forced-circulation evaporator is quite different, both in design and in operation.

More directly connected with evaporators is the study made by Hausbrand (3) in 1899. The discussion was entirely from a theoretical standpoint and was not supplemented by any direct experimental data. It deals with the effect of currents of vapor upon droplets of liquor under various circumstances and gives equations expressing the relationship for each case.

Of the various cases that Hausbrand considered, one in which a vertical current of steam meets a drop of liquid

The entrainment in a laboratory forced-circulation evaporator, containing thirty tubes, 0.5 inch o. d. and 8 feet long, is determined by boiling a salt solution and testing for sodium chloride in the condensed vapors.

An attempt is made to analyze the problem mathematically. The treatment, though incomplete and very approximate, indicates a type of function that should be a measure of the entrainment. The results approximately confirm these deductions, although almost as good correlation will be obtained by considering that entrainment is proportional to mass velocity.

thrown obliquely is similar to the present problem; although he was mainly interested in the point at which entrainment begins rather than the amount of entrainment, his treatment of the problem might easily be modified to apply to this case.

Hausbrand found the following relationships:

$$D = \psi \gamma_v Q \frac{v^2}{2g}$$
$$\frac{D}{G} - 1 = \frac{2c}{gt} \left(\sin \alpha - \cos \alpha \tan \frac{\alpha}{2} \right)$$

where D = pressure exerted upon the drop by the vapor
 Q = projected plane surface of the drop
 γ_v = density of the vapor
 v = relative velocity between vapor and drop
 g = acceleration of gravity
 ψ = a numerical coefficient
 c = initial velocity of drop
 α = angle between path of the drop and the horizontal
 t = time the drop is in vapor current before hitting the wall
 G = diameter of drop

These equations when combined yield a relationship similar to that derived below.

DISCUSSION OF ENTRAINMENT

Entrainment is entirely a mechanical process. A small portion of the liquor in the body of the evaporator is carried upward in the form of droplets by the vapor, and passes into the condenser, resulting in a loss of the material con-

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certain point will be carried into the vapor space, while those passing outside this point will strike the wall of the evaporator. The point defined is, therefore, the point through which the largest particles to be carried over will pass and is at a certain fraction of the distance between the deflector and the wall. This fraction is unknown but is assumed to be constant for a given evaporator.

Notwithstanding these simplifying assumptions, the relationship between some of the factors is quite involved, and it is necessary to neglect their effect in order to avoid a complicated and practically useless equation. It should be understood, therefore, that the expression derived is not entirely rigorous and is not presented as an exact theoretical equation. This derivation is merely a line of reasoning based upon various logical assumptions and was undertaken to ascertain the various factors that influence the amount of entrainment and to determine, if possible, the manner in which these factors should be grouped in the equation.

In Figure 2 a cross-sectional diagram of the curtain and the forces upon it is shown. On the space diagram the line $\overline{WX}$ is the tip of the deflector making an angle, α , with the horizontal line $\overline{XO}$. $\overline{OL}$ is the wall of the evaporator at a distance S from $\overline{X}$. $\overline{XL}$ denotes the path of the curtain if free from accelerating effects. M is that point at which the largest particles to be carried over will pass the horizontal. All the particles passing to the left of this point will travel up into the vapor space, while those passing to the right of M will strike the wall.

The forces acting upon each droplet are those of gravity, F_g , acting downward and friction from the vapor, F_v , acting at an angle, β , with the vertical. The resultant of the two forces is the accelerating force, F_a , at an angle, γ , with the vertical. In comparison with the accelerating force, the force of gravity will be very small and will be neglected in this derivation, so that the force exerted by the vapor will be considered to be equal to the accelerating force acting at an angle, γ , with the vertical.

The motion of a droplet following the resultant (not actual) path $\overline{XM}$ can be divided into components, $\overline{XZ}$ in the direction of the original velocity and $\overline{ZM}$ in the direction of the accelerating force; or, briefly, the accelerating force, F_a , must be sufficient to cause the drop to move through the distance $\overline{ZM}$ in the time it would have traveled from X to Z if not so affected.

If C is the original velocity, and t the time for passing from X to Z ,

$$t = \frac{\overline{XZ}}{C}$$

and the acceleration required is

$$a = \frac{2C^2\overline{ZM}}{\overline{XZ}^2}$$

The mass of the droplet is $A\rho'D^3/g$,

$$\text{therefore } F_a = \frac{2A\rho'D^3C^2\overline{ZM}}{g\overline{XZ}^2}$$

Since $\overline{OX}$ equals S and $\overline{XM}$ is defined as $f(OX)$,

$$\overline{XM} = f(S)$$

$$\overline{YM} = f(S) \sin \alpha$$

The angle $(\phi + \theta) = (\gamma - 90)$
 $\phi = (90 - \alpha)$

$$\theta = (\gamma - 90) - (90 - \alpha) = (\alpha + \gamma - 180)$$

$$\begin{aligned}\overline{ZM} &= \frac{\overline{YM}}{\cos(\alpha + \gamma - 180)} \\ &= \frac{f(S) \sin \alpha}{-\cos(\alpha + \gamma)}\end{aligned}$$

$$\overline{XZ} = \overline{XY} + \overline{YZ}$$

$$\begin{aligned}\overline{XZ} &= f(S) \cos \alpha + f(S) [\sin \alpha \tan(\alpha + \gamma)] \\ &= f(S) [\cos \alpha + \sin \alpha \tan(\alpha + \gamma)]\end{aligned}$$

Substituting above,

$$F_a = \frac{2A\rho'D^3C^2(\sin \alpha)}{gf(S) [-\cos(\alpha + \gamma)] [\cos \alpha + \sin \alpha \tan(\alpha + \gamma)]^2}$$

For any given evaporator, α , γ , S , and $f(S)$ are assumed to be constant.

$$F_a = R\rho'D^3C^2$$

where R = a constant

Since F_g is neglected and F_v equals F_a ,

$$F_v = R\rho'D^3C^2$$

The expression for the force due to friction between a particle and a fluid is (1):

$$F = \frac{\psi D^2 u^2 \rho}{g} \phi \left(\frac{D' u \rho}{z} \right)$$

In this expression u is defined as the velocity of the fluid relative to the particle. Although the droplets being considered here are in motion, the direction of that motion is nearly at right angles with that of the vapor and under the assumptions made above would not affect the force exerted by the vapor in the direction of acceleration. The velocity of the vapor through the curtain, u_v , is therefore substituted for u .

Setting the expressions for the forces equal gives the equation:

$$\frac{\psi D^2 u_v^2}{g} \phi \left(\frac{D' u \rho}{z} \right) = R\rho'D^3C^2$$

Solving for D ,

$$D = \frac{\psi u_v^2 \rho \phi \left(\frac{D' u \rho}{z} \right)}{g R \rho' C^2}$$

The droplets carried by the vapor will range in diameter from D , the diameter of the largest, to zero. The number of drops of each diameter will undoubtedly follow some function of the probability curve and will change with the densities of the vapor and liquid. The volume of liquid carried per cubic foot of vapor will, therefore, be some function of D , ρ , and ρ' . At the present time it is impossible to determine the nature of this function without considerable study outside the scope of the present problem. It has been assumed, therefore, that it can be empirically expressed as:

$$V_L = b \left(\frac{\rho}{\rho'} \right)^\delta \cdot \lambda(D)$$

where V_L = volume of liquid per million cu. ft. of vapor
 b = an unknown constant
 δ = an unknown power
 $\lambda(D)$ = an unknown function of D

If E equals the pounds of liquid carried per million pounds of vapor,

$$E = \frac{b\rho'\rho^\delta\lambda(D)}{\rho\rho'^\delta}$$

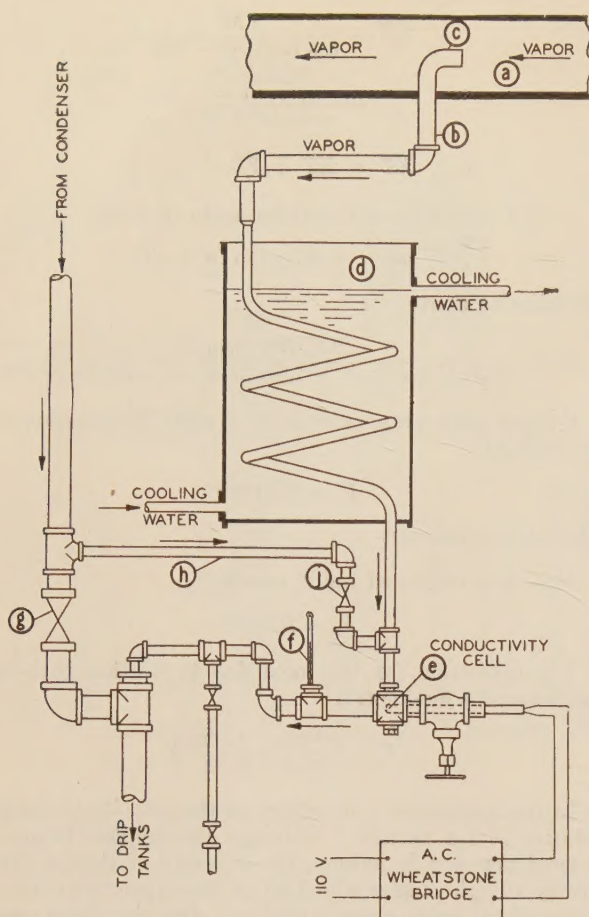


FIGURE 3. APPARATUS FOR ANALYSIS OF OVERHEAD DRIPS

Substituting for D ,

$$E = \frac{b\rho^{\delta-1}}{\rho'^{\delta-1}} \cdot \lambda \left[\frac{\psi u_v^2 \rho \phi \left(\frac{D'u\rho}{z} \right)}{gR\rho'C^2} \right]$$

In the range covered, $\phi \left(\frac{D'u\rho}{z} \right)$ will be practically constant; and since ρ is very small in comparison with ρ' it may be assumed that $(\rho' - \rho) = \rho'$. Then including b and ψ in the λ' function,

$$E = \frac{(\rho)^{\delta-1}}{(\rho')^{\delta-1}} \cdot \lambda' \left(\frac{\rho u_v^2}{\rho'C^2} \right)$$

EXPERIMENTAL PROCEDURE

The data for the study of this problem were obtained upon an experimental, small-tube, forced-circulation evaporator. The general method used to determine the amount of liquid carried with the vapor was as follows: A solution of sodium chloride, approximately 10 per cent by weight, was placed in the body of the evaporator, and distilled water was added continuously to replace that which was evaporated and thus maintain a constant concentration. The overhead condensate and the solution in the evaporator were analyzed, and from their relative concentrations the pounds of liquid carried per million pounds of vapor were calculated.

The general construction of the apparatus is shown in Figure 1.

It consists of the evaporator equipped with all the necessary accessories, such as the circulating pump, p , surface condenser, l , vacuum pump, r , manometer tubes, q , and calibrated tanks, s

and t , for the steam and overhead condensate. It is essentially standard in design so that only those parts which are involved in the taking of data for this research will be discussed more fully.

The heating element is a double-walled, plate-steel cylinder, 18 inches in diameter, with Sil-O-Cel packed in the annular space. The tube sheets are 0.75 inch thick and are 8 feet apart. The heating surface consists of thirty 0.5-inch o. d. 20-gage copper tubes giving an area of 27.02 square feet.

Steam is introduced into the heating element at the center of the top tube sheet, and the condensate is removed at a point near the circumference of the bottom tube sheet. The vent can be connected to the atmosphere or to the vacuum pump as desired.

Mercury manometers are used to measure steam pressure, vacuum, pressure drop across the tubes, and pressure drop across an orifice in the circulation line.

Since the main condenser has a rather large surface and retains a considerable amount of liquid, the change in the concentration of the liquid leaving is gradual so that an appreciable time is required to reach equilibrium after changing conditions. In order to expedite the taking of data, the accessory apparatus shown in Figure 3 was installed.

The horizontal section of the 6-inch vapor line between the top of the evaporator and the condenser is shown at a . The 1-inch pipe, b , is welded into the bottom of a and projects far enough inside so that the short nipple, c , is directly in the center of a . Pipe b is connected to the auxiliary condenser, d , by a horizontal 1-inch pipe.

Auxiliary condenser d is made up of a 0.75-inch copper coil submerged in a steel tank filled with water. The water is maintained at such a level that approximately 15 feet of the coil are submerged. The necessary heating surface was determined to be such that the effective area has the same relationship to the area of the main condenser that the cross-sectional area of pipe c has to the cross-sectional area of the annular space between c and a . Assuming that the product of the over-all heat transfer coefficient and the temperature drop is approximately the same in the main and auxiliary condensers, such a quantity of vapor will be condensed in condenser d that the velocity of the vapor through pipe c will be the same as the mean velocity through the annular space. In this way excessive turbulence at this point and the loss of entrained particles is avoided.

The condensate formed in condenser d runs directly down into a small trap and passes the conductivity cell, e , inserted in the side arm of a cross. The liquid comes into contact with the thermometer, f , placed in a tee in the bottom of the trap and then leaves the trap, dropping into the top of a tee in the discharge line from the main condenser.

By partially closing valve g in the main condensate line and opening valve i , it is possible to run a portion of the condensate from the main condenser through pipe h to the trap whenever desired.

In the preliminary runs the use of this by-pass showed that after the period of lag the condensate from the two condensers was of approximately the same composition. However, to insure that the data taken were representative, the auxiliary condenser was used only at the start of the run until equilibrium had been reached in the main condenser, and then the by-pass was opened and liquid from the main condenser used during the remainder of the run.

As was stated above, the amount of entrainment was determined from the relation between the concentration of sodium chloride in the body of the evaporator and in the overhead condensate. The concentration of the solution within the evaporator was determined by titrating with standard silver nitrate, using potassium chromate as an indicator.

The overhead condensate was analyzed by a Leeds and Northrup conductivity cell. This was calibrated against salt solutions of known concentration over the range from 1 to 100 p. p. m. of sodium chloride, and at temperature intervals of 5° C. (9° F.).

Successive runs, 20 minutes in length, were made under each set of conditions until a check was obtained (usually

two were sufficient). The level of the liquid in the steam and overhead condensate tanks was read at the start and at the end of each run; and ten readings, at intervals of 2 minutes, were taken on the four manometers, the Wheatstone bridge, and the thermometer in the trap. Qualitative notes were also made upon the condition of the curtain, and the vertical distance from the tip of the deflector to the point the curtain met the wall was measured. These measurements were approximate because the sight glasses were some distance apart, and it was therefore necessary to estimate the lower point.

Between periods of operation there was a considerable change in the sodium chloride concentration in the body due to loss of the liquor, but while the evaporator was running the change was very small. A sample of the liquor was removed for analysis at the beginning and end of each run. Since they were of approximately the same concentration, the average of the two was used in the calculation of all the runs made during the period.

CORRELATION OF DATA

The data obtained during the runs had to be converted into a form in which they could be correlated. The principal items desired for this study were E , the entrainment; u_v , the velocity of the vapor through the curtain; u_L , the velocity of the liquid through the tubes; ρ , the density of the vapor; and ρ' , the density of the liquid.

The difference between the saturation temperatures of steam and vapor, calculated from the manometer readings, was taken as the temperature drop through the heating surface. Since there was a boiling point rise due to the presence of sodium chloride in the liquor, this is the apparent rather than the true temperature drop.

The readings on the Wheatstone bridge and the thermometer could not, of course, be averaged directly. Instead, the concentration of the condensate in parts per million at the

time of each reading was taken from the calibration curves, and the ten concentrations were averaged to obtain a value representative of the entire run. If the average concentrations of the condensate and of the liquor are known:

$$E = \frac{(\text{concn. of condensate}) 100}{\text{concn. of liquor}}$$

The vapor velocity, u , could be determined from the amount of liquid collected in the overhead condensate tanks during the run. Expressed algebraically,

$$u_v = \frac{(\Delta d) (W) (v_v)}{(20) (60) A_c}$$

where Δd = difference in level of overhead condensate, mm.

W = calibration factor of tank, lb./mm.

v_v = specific volume of vapor, cu. ft./lb.

A_c = area of curtain, sq. ft.

In calculating the area of the curtain to be used in this equation, the effect of gravity was neglected and the curtain assumed to be straight rather than parabolic in shape. In each case the area was determined from the observed length.

Since the pressure difference across the orifice as indicated by the ten manometer readings varied only slightly, the readings could be averaged directly and the corresponding liquor velocity through the tubes, u_L , determined from the calibration curve. The density of the vapor, ρ , is the reciprocal of the specific volume used in the calculation of vapor velocity, and that of the liquor, ρ' , was obtained from the International Critical Tables giving the density of sodium chloride at various temperatures.

A composite summary of all the data which were essential for the correlation is shown in Table I.

The data taken during the experimental runs were rather difficult to correlate for several reasons. In the first place, there had been no previous study of entrainment in an evaporator of this type to give any idea of the grouping of the

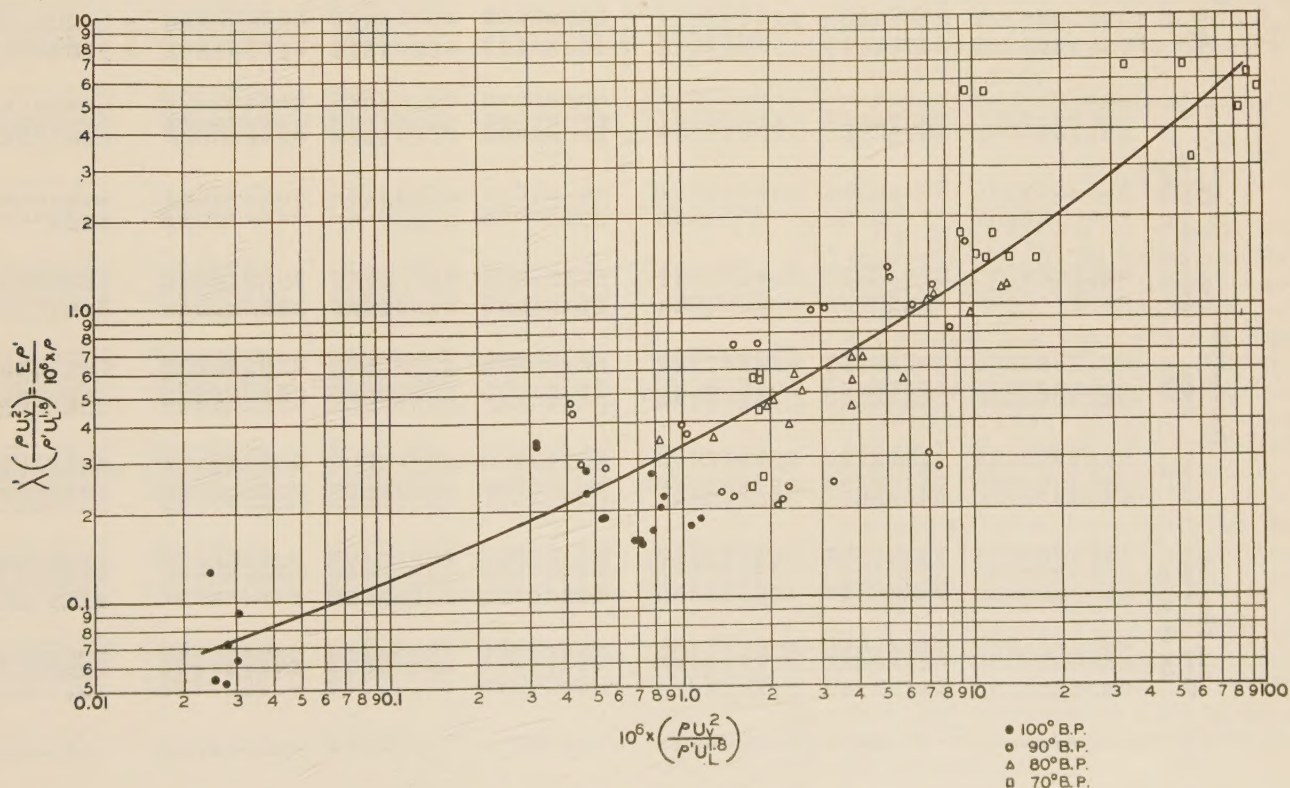


FIGURE 4. PLOT OF ENTRAINMENT FACTOR

100° C. = 212° F.; 90° C. = 194° F.; 80° C. = 176° F.; 70° C. = 158° F.

then

$$\frac{E}{\rho^{1.5}u_v} = K \frac{1}{u_L^{2m'}/4}$$

When $E/\rho^{1.5}u_v$ is plotted against u_L^2 , the most reliable points form a band with a slope determined to be -0.45 . If $m'/4 = 0.45$; m' is 1.8 and can be substituted in the original expression:

$$E = \frac{\rho}{\rho'} \cdot \lambda' \left(\frac{\rho u_v^2}{\rho' u_L^{1.8}} \right)$$

or

$$\frac{E\rho'}{\rho} = \lambda' \left(\frac{\rho u_v^2}{\rho' u_L^{1.8}} \right)$$

The density of the liquid, ρ' , is again included in the equation purely on the basis of the derivation and dimensional analysis. Since it did not vary appreciably at any time, there is no experimental proof that it should be a factor.

Although various methods were used to determine the nature of the λ' function, it did not resolve itself into any of the simpler relationships and for this reason it was considered more desirable to leave the equation in the above form and express the function as a plot.

The plot of

$$\frac{\rho u_v^2}{\rho' u_L^{1.8}} \text{ vs. } \frac{E\rho'}{\rho} \text{ which equals } \lambda' \left(\frac{\rho u_v^2}{\rho' u_L^{1.8}} \right)$$

is shown in Figure 4. The points fall into a band which may include a series of parallel lines because of the influence of

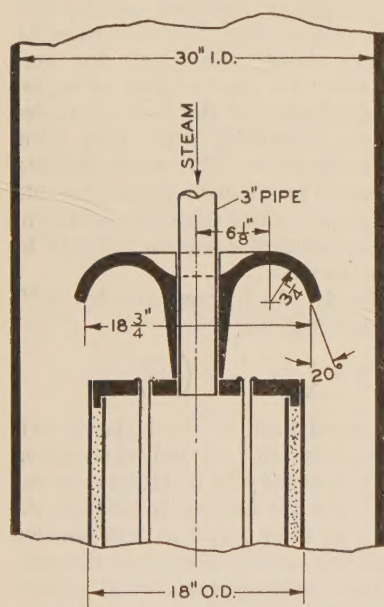


FIGURE 5. DETAIL OF DEFLECTOR AND CURTAIN

only a single row of 0.5-inch tubes giving a very light curtain, easily affected by slight fluctuations; thus, the entrainment varied irregularly as the structure of the curtain changed. Although the variation in this case was, therefore, quite large because of the lightness of the curtain, and a heavier curtain would undoubtedly narrow the band, the existence and magnitude of such a variable cannot be proved because the only experimental means of detecting any difference in the curtain is by qualitative observation.

In those runs where the volume of vapor passing out of the tubes was comparatively large, the whole edge of curtain was blown upward by the force of the vapor. This occurs whenever the vapor velocity passes a certain critical value, definite

for each evaporator. The amount of entrainment under these circumstances is very great and does not follow the same relationship that it does when the vapor velocity is below the critical. Accordingly, the points on the plot corresponding to these runs are much higher than the average and are considerably out of line.

There are some indications that the amount of liquid in the curtain has an influence. The points of a few of the series lie in the same relative order as the liquor velocities; but in the majority of cases other indeterminate factors evidently enter and alter the relationship so that no conclusions can be drawn in this regard. A single line has, therefore, been drawn through the points to give an average value for the entrainment factor.

CONCLUSION

As a result of this research, it can be concluded that the amount of entrainment in a forced-circulation evaporator when using a nonfoaming liquid is given by the equation:

$$E = \frac{\rho}{\rho'} \cdot \lambda' \left(\frac{\rho u_v^2}{\rho' u_L^{1.8}} \right)$$

The λ' function will, in general, follow the form of the plot shown in Figure 4. The actual values given for the function, however, apply only for a liquid with a surface tension and viscosity essentially the same as those of water and for an evaporator whose dimensions in the vicinity of the deflector have the same relation as those of the evaporator used in this study and shown in Figure 5.

ACKNOWLEDGMENT

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NOMENCLATURE

E	= entrainment, lb. liquor per million lb. vapor
ρ	= density of vapor, lb. per cu. ft.
ρ'	= density of liquor, lb. per cu. ft.
D	= diameter of largest droplets
F_g	= force exerted by gravity
F_v	= force exerted by vapor
F_a	= accelerating force
u_v	= velocity of vapor through curtain, ft. per sec.
u_L	= velocity of liquor through tubes, ft. per sec.
C	= velocity of liquor leaving deflector, ft. per sec.
S	= distance from deflector tip to wall
α	= angle deflector makes with horizontal
β	= angle vapor makes with vertical (assumption 2)
γ	= angle accelerating force makes with vertical
v_L	= volume of liquor carried per million cu. ft. vapor
v_v	= volume of vapor leaving tubes, cu. ft. per sec.
g	= acceleration of gravity
$\left(\frac{D'u\rho}{z} \right)$	= Reynolds' criterion for the vapor
$A, B, k, m, b, \psi, \delta$	= constants

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